YU ISSN 0015-3206
FZKAAA 23 (2) 107 (1991)

ALPHA-DECAY WIDTHS OF LEAD AND SUB-LEAD NUCLEI

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Received 24 August 1990

Revised manuscript received 5 May 1991

UDC 539.164

Original scientific paper

The non-local potential with Woods-Saxon form as static part has been applied here to calculate the reduced widths of some lead and sub-lead nuclei. The results are quite consistent with those previously found using Igo's form of potential as static part.

1. Introduction

The alpha-decay width has been one of the most important informative factor about nuclear structure. Recently much interest has been aroused[1-4] in the calculations of α-decay widths. The semi-empirical value of α-reduced width given by

$$\delta_l^2 = hF_l \tag{1}$$

is obtained from

$$\lambda_l = F_l \times P_l \tag{2}$$

where F_l is the total internal transition probability, P_l is the penetration factor and λ_l the partial decay constant. As none of F_l and P_l is measurable, we are to make assumptions on any one of them. We prefer to make assumptions on P_l

as F_l contains a number of unknown parameters. In earlier works[5-7] P_l was calculated on the basis of a non-local α-nucleus potential and the penetration ratios $P_l/P_{l'}$, (where l' refers to maximum intensity α-groups) were found to accomodate the intensity ratios to a large extent as compared to the observed intensity ratios for rare earth, trans-lead and trans-uranium nuclei. On the other hand the α-intensity ratios calculated using static potentials[8-12] show large deviations from the observed values. Considering this in view, the value of P_l obtained with the aforesaid non-local potential was supposed to be more realistic in evaluating F_l from Eq. (2) and reduced width[13-14] from Eq. (1). In these works, Igo's potential was used as static part whereas Woods-Saxon form of potential has generally been used in discussing scattering problems. It is however not clear, when the same nuclear interaction is involved, why the two areas, namely α-decay and scattering, should have different optical model potentials for the static part. In view of this, Woods-Saxon form of potential was recently used[15-16] as static part in the non-local α-nucleous potential and the calculated values of $P_l/P_{l'}$, were found to be in good agreement with the observed intensity ratios for rare-earth and trans-lead nuclei. The method has now[17] been extended to calculate the reduced transition probability for a number of trans-uranium spheroidal nuclei and the results are in excellent agreement with the experimental values. It is also interesting to note that almost same set of W. S. parameters (with minor variations for different regions) works for the entire range of nuclei (from rare-earth to trans-uranium) unlike their wide variations for different nuclei in scattering. It is also significant that use of local potential in α-decay[1,18,19] requires adjustment of parameters in individual cases whereas with non-local α-nucleus potential, same set of parameters for wide ranges of nuclei[15-17] yield satisfactory agreement with observed intensity ratio. The reason for uniform set of parameters in α-decay may perhaps be attributed to the non-local character of the potential.

The purpose of the present work is to calculate the reduced width of some lead and sub-lead nuclei by using the aforesaid non-local α-nucleus potential with Woods-Saxon potential as static part. We consider here the ground to ground state transitions.

2. *Penetrability factor*

For the spherically symmetric non-local potential the interaction kernel as defined earlier[5] is given by

$$J(\vec{r}, \vec{r}') = V(r)\, \delta_b(\vec{r} - \vec{r}') \tag{3}$$

where

$$\delta_b(\vec{r} - \vec{r}') = \pi^{-3/2} b^{-3} \exp[-(\vec{r} - \vec{r}')^2/b^2]. \tag{4}$$

As already stated, we take the static part $V(r)$ to be of Woods-Saxon form viz.

$$V(r) = -V_0/\{1 + \exp[(r - r_0 A^{1/3})/d]\} \tag{5}$$

where r, r_0 and d are in fm and V_0 is in MeV.

Using the formalism developed in a previous paper[5] we obtain the penetration factor as

$$P_l = \exp\left[-2\left\{(2\mu)^{1/2}/\hbar \int_{R_l}^{R_0} \left(\frac{2(Z-2)e^2}{r} + \frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2} - \right.\right.\right.$$

$$\left.\left.\left. - V_{\pm} f(r) K_l(r) - E\right)^{1/2} dr\right\}\right] \tag{6}$$

where

$$V_{\pm} = V_0 [S + (1 - S)(-1)^l], \qquad V(r) = -V_0 f(r) \tag{7}$$

$$K_l(r) = \frac{1}{2}[1 + \mathrm{erf}(z)]\left(1 - \frac{l(l+1)b^2}{4r^2}\right) \tag{8}$$

$$z = \frac{r - R_l}{b} \tag{9}$$

$$R_0 = \frac{2(Z-2)e^2}{E}. \tag{10}$$

The subscript $\pm$ corresponds to l even and odd, respectively., S is the mixture proportion of the ordinary part of the potential, b is the range of non-locality, E is the decay energy after being corrected for nuclei recoil and electron screening, and R_l is the inner turning point.

From Eqs. (1) and (2), the reduced width δ_l^2 is given by

$$\delta_l^2 = h \times \frac{\lambda_l}{P_l}. \tag{11}$$

3. Results

The value of the penetration factor P_l has been calculated from Eq. (6) by using a modified Simpson's rule with 160 strips. The W. S. parameters used here are $V_0 = 210$ MeV, $r_0 = 1.37$ fm, $d = 0.5$ fm, the same as used earlier[15] for heavy nuclei. As before, we have taken the range of non-locality b to be 0.7 fm and the mixture proportion S to be 60%.

The values of P_l and the corresponding reduced width δ^2 (Eq. 11) (for a number of lead and sub-lead nuclei) have been presented in table along with the inner turning points, partial α half lives and the decay energies. The values of the reduced widths corresponding to non-local α-nucleus potential with Igo's potential as static part have been presented in column 8 for sake of comparison.

TABLE 1.

Nucleus	l of alpha	Partial α-half life/s	Decay energy E (MeV)	Penetrability factor P_l	R_l/f_m	Reduced width N. W.	(Eq. 11)* N. I.
1	2	3	4	5	6	7	8
$^{167}_{76}Os^{a)}$	0	1.05	5.984	0.8131(−20)	0.8229(−12)	0.336	0.217
$^{169}_{76}Os^{a)}$	2	30.9	5.707	0.3311(−21)	0.8251(−12)	0.280	0.190
$^{168}_{76}Os^{a)}$	0	4.48	5.814	0.1697(−20)	0.8238(−12)	0.378	0.248
$^{170}_{76}Os^{a)}$	0	59.2	5.535	0.1119(−21)	0.8260(−12)	0.434	0.291
$^{170}_{78}Pt^{a)}$	0	0.006	6.605	0.2741(−18)	0.8275(−12)	1.745	1.660
$^{172}_{78}Pt^{a)}$	0	0.090	6.459	0.9075(−19)	0.8301(−12)	0.351	0.243
$^{174}_{78}Pt^{a)}$	0	0.900	6.169	0.7850(−20)	0.8322(−12)	0.406	0.286
$^{178}_{78}Pt^{b)}$	0	264.0	5.582	0.2938(−22)	0.8364(−12)	0.370	0.274
$^{180}_{78}Pt^{c)}$	0	17.3(+3)	5.376	0.3389(−23)	0.8388(−12)	0.049	0.037
$^{182}_{78}Pt^{c)}$	0	78.0(+4)	5.034	0.6339(−25)	0.8407(−12)	0.058	0.045
$^{184}_{78}Pt^{c)}$	0	10.3(+3)	4.692	0.7512(−27)	0.8427(−12)	0.037	0.029
$^{186}_{78}Pt^{c)}$	0	51.4(+8)	4.418	0.1402(−28)	0.8463(−12)	0.040	0.030
$^{179}_{80}Hg^{c)}$	0	2.05	6.560	0.4115(−19)	0.8391(−12)	0.034	0.025
$^{183}_{80}Hg^{c)}$	0	73.3	6.031	0.4115(−21)	0.8434(−12)	0.095	0.075
$^{185}_{80}Hg^{c)}$	2	960.0	5.615	0.3363(−23)	0.8450(−12)	0.889	0.725
$^{187}_{80}Hg^{c)}$	0	13.2(+5)	5.257	0.1076(−24)	0.8483(−12)	0.020	0.015
$^{180}_{80}Hg^{d)}$	0	7.25	6.257	0.2956(−20)	0.8395(−12)	0.134	0.102
$^{182}_{80}Hg^{d)}$	0	126.0	5.996	0.2756(−21)	0.8417(−12)	0.083	0.064
$^{184}_{80}Hg^{c)}$	0	2350.0	5.783	0.3564(−22)	0.8441(−12)	0.034	0.027
$^{186}_{80}Hg^{c)}$	0	52.5(+4)	5.320	0.5159(−24)	0.8471(−12)	0.025	0.019
$^{187}_{82}Pb^{d)}$	2	850.0	6.212	0.1762(−21)	0.8493(−12)	0.019	0.014
$^{189}_{82}Pb^{d)}$	0	11.9(+3)	5.853	0.9781(−23)	0.8509(−12)	0.025	0.018
$^{191}_{82}Pb^{c)}$	0	60.0(+4)	5.518	0.2755(−24)	0.8526(−12)	0.017	0.014
$^{186}_{82}Pb^{d)}$	0	329.0	6.458	0.3165(−20)	0.8471(−12)	0.003	0.002
$^{188}_{82}Pb^{d)}$	0	742.0	6.120	0.1386(−21)	0.8504(−12)	0.028	0.021
$^{190}_{82}Pb^{f)}$	0	80.0(+2)	5.710	0.2240(−23)	0.8518(−12)	0.160	0.129
$^{192}_{82}Pb^{g)}$	0	36.8(+5)	5.220	0.8100(−26)	0.8530(−12)	0.096	0.091

Figures in brackets in columns (3), (5) and (6) denote powers of 10. N. W, N. I refer to the non-local potential taking for the static part the Wood-Saxon potential and Igo's potential, respectively. a) Ref. 22 b) Ref. 23 c) Ref. 24 d) Ref. 20 f) Ref. 25 g) Ref. 26.

4. *Discussion*

It can be seen from column 7 and 8 of the table that the values of reduced widths calculated in the present work are in good agreement (within 40%) with the corresponding values calculated previously (cf. Ref. 13) using Igo's potential as static part. A fixed set of W. S. parameters viz. $V_0 = 210$ MeV, $r_0 = 1.37$ fm, and $d = 0.5$ fm works successfully for all sub-lead, lead and trans-lead nuclei.

This has been possible by using non-local effects with the W. S. potential. Like scattering, adjustment of parameter in individual cases is required in α-decay also if one uses only the local potential as mentioned in the introduction. So our findings here strengthens our earlier view (cf. Refs. 15, 16) that W. S. potential can provide a suitable alternative to Igo's potential as static part of the non-local α-nucleus potential.

From the table, it can be seen that the usual trend of decreasing reduced widths with increasing neutron number is reproduced for most of the nuclei with few exceptions. ^{167}OS, ^{169}OS follows this trend whereas the values of reduced widths of ^{168}OS and ^{170}OS are almost same. The values of reduced widths for odd-A OS isotopes are smaller than the even-A isotopes as expected. Moreover all the Pt-isotopes follows the usual trend, the values of hF_1 for pairs (^{172}Pt, ^{174}Pt) (^{180}Pt, ^{182}Pt) and (^{184}Pt, ^{186}Pt) being nearly equal. For Hg-isotopes we find that the reduced widths decrease with increase of neutron number for even A isotopes whereas the trend is broken in case of ^{179}Hg, ^{183}Hg and ^{185}Hg. Reduced width of ^{186}Hg is somewhat smaller and this is perhaps due to the fact that α-decay in it connects two different neutron levels ($f_{5/2} \rightarrow p_{3/2}$). Similar is the case for Pb isotopes but there is greater anomaly in these nuclei. The isotopes ^{186}Pb, ^{188}Pb do not follow the usual trend where as the trend is maintained by ^{190}Pb and ^{192}Pb. Also the reduced widths of odd-A isotopes are nearly same. It may be noted that hF_l for ^{186}Pb, ^{188}Pb have been calculated by using data which were obtained by estimation from the cross sections of their productions from heavy ion induced reactions rather than direct experiment[20]. Toth et al.[21] have shown that the anomalous behaviour of ^{188}Pb is probabily due to incorrect data. There has been speculation that anomalous behaviour of Pb and Hg isotopes is caused by change in their nuclear structure, but no theory yet could reconcile this behaviour. The low values of hF_l for Pb isotopes are compatible to our expectation that the reduced widths should be low corresponding to a major closed shell at $Z = 82$.

5. *Conclusion*

(i) It is found that the reduced widths with non-local barrier with W. S. potential as static part are in good agreement with their corresponding values obtained with Igo's potential as static part. This shows that W. S. potential can provide an alternative to Igo's potential in α-decay theory provided non-local effects are taken into account.

(ii) Except for a few nuclei, the trend of decreasing reduced width with increasing neutron number is reproduced.

Acknowledgement

The authors gratefully acknowledge the computer facilities extended by Indian School of Mines, Dhanbad. The facilities extended by Saha Institute of Nuclear Physics, Calcutta and Indian Association of Cultivation of Science, Calcutta are also acknowledged with thanks.

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ŠIRINE ALFA RASPADA JEZGARA OLOVA I LAKŠIH OD OLOVA

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UDK 539.164

Originalni znanstveni rad

Neolokalni potencijal Wood--Saxonove forme kao statički dio primijenjen je za račun reduciranih širina nekih jezgara olova i lakših od olova. Rezultati su u dobrom suglasju s ranije nađenima uz upotrebu Igo-ove forme statičkog dijela potencijala.