YU ISSN 0015—3206
FZKAAA 21 (3) 297 (1989)

THE GLUON SELF-ENERGY IN THE LIGHT-CONE GAUGE AND THE PIGUET-SIBOLD IDENTITY

ANĐELKA ANDRAŠI

Ruđer Bošković Institute, 41001 Zagreb, P. O. B. 1016, Croatia, Yugoslavia

Received 23 March 1989

UDC 539.12.01

Original scientific paper

We discuss the dependence of the complete gluon self-energy tensor on the second light-like vector in the light-cone gauge. The Piguet-Sibold identities are verified.

1. Introduction

The light-cone gauge has become increasingly popular with the revival and intensive research of string theories. The gauge is not only ghost-free, but can also be used to remove from the theory all unphysical degrees of freedom associated with gauge transformations[1]. It is also the only gauge in which a quantum formulation of superstring theories is known. However, there are problems about defining axial gauges carefully. The most convenient way of defining the light-cone gauge seems to be by introducing two light-like vectors n and n^*, satisfying

$$n^2 = n^{*2} = 0, \qquad n \cdot A \equiv 0. \tag{1}$$

The prescription for the gluon propagator after Mandelstam[2] is

$$G_{\mu\nu} = \frac{1}{k^2 + \mathrm{i}\varepsilon}\left(-\delta_{\mu\nu} + \frac{k_\mu n_\nu + k_\nu n_\mu}{n \cdot k + \mathrm{i}\eta n^* k}\right) \tag{2}$$

and after Leibbrandt is[3]

$$G_{\mu\nu} = \frac{1}{k^2 + \mathrm{i}\varepsilon}\left[-\delta_{\mu\nu} + \frac{(k_\mu n_\nu + k_\nu n_\mu)\, n^* \cdot k}{n \cdot k\, n^* \cdot k + \mathrm{i}\eta}\right]. \tag{3}$$

Gauge-invariant quantities should be independent of the choice of n^* as well as n. This invariance is guaranteed by the Piguet-Sibold identity[4]

$$\Delta\Gamma = \Gamma * \Gamma'. \tag{4}$$

The infinitesimal change in the generating functional of one-particle-irreducible vertices $\Delta\Gamma$ due to an infinitesimal change Δn^* in n^* is given by the BRST convolution[5] of the generating functional with the functional containing the extra vertex

$$\bar{c}\, \Delta N_\mu (\partial)\, A_\mu, \tag{5}$$

where $N(A)$ is the functional in the gauge-fixing term and $\bar{c}$ the antighost field

$$\lim_{\alpha \to 0} \frac{1}{2\alpha} [N \cdot A]^2. \tag{6}$$

Mandelstam's prescription corresponds to the choice for N[6] in (6):

$$N_\mu = n_\mu + \mathrm{i}\eta n^*_\mu \tag{7}$$

and so the ghost propagator is

$$(n \cdot k + \mathrm{i}\eta n^* k)^{-1} \tag{8}$$

and the ghost-gluon vertex is

$$n_\mu + \mathrm{i}\eta n^*_\mu. \tag{9}$$

Leibbrandt's prescription corresponds to the gauge-fixing term[6]

$$N \cdot A = n \cdot A + \mathrm{i}\eta\, (n^* \partial)^{-1}\, \Box^{-1} \partial A, \tag{10}$$

with the ghost propagator

$$(n \cdot k\, n^* \cdot k + \mathrm{i}\eta)^{-1} \tag{11}$$

and the vertex

$$n^* \cdot k\, n_\mu + \mathrm{i}\eta\, (k^2)^{-1}\, k_\mu. \tag{12}$$

According to Piguet and Sibold, to ensure the gauge independence, the gluon self-energy tensor to one-loop order has to satisfy the identity

$$\Delta\pi(x, y) = \frac{{}'\delta^2 \Delta\Gamma}{\delta A(x)\,\delta A(y)} = \frac{\delta^2\Gamma}{\delta A(x)\,\delta A(z)} \cdot \frac{\delta\Gamma'}{\delta u(z)\,\delta A(y)} + (x \leftrightarrow y), \tag{13}$$

where $u(z)$ is the source in Γ'

$$u_\lambda D_\lambda c. \tag{14}$$

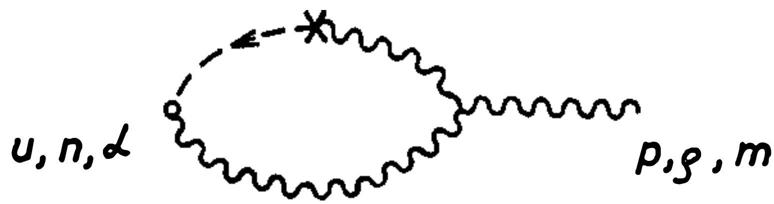
When we Fourier transform this, we get

$$\Delta\pi_{\mu\nu}(p) = (p_\mu p_\lambda - \delta_{\lambda\mu} p^2)\,\Gamma'_{\lambda\nu} + (p_\nu p_\lambda - \delta_{\lambda\nu} p^2)\,\Gamma'_{\mu\lambda}. \tag{15}$$

2. *Complete Γ' to one-loop order*

Here we calculate the ultraviolet divergent part (in dimensional regularization) as well as the finite part of the two-point function under the change in Δn^*,

$$\frac{\delta\Gamma'}{\delta u_\lambda(x)\,\delta A_\mu(y)}.$$



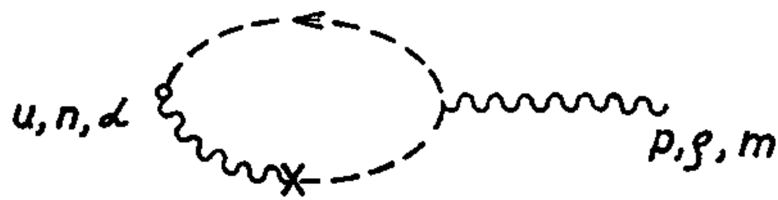


Fig. 1. The two graphs contributing to Eq. (16). The cross signifies the action of the term (5), and the circle indicates the action of the source (14). Wavy lines are gluons and dotted lines represent ghosts.

There are two graphs, shown in Fig. 1, which contribute, respectively,

$$\Gamma'(1) = \eta g^2 C_{YM} \delta_{mn} \int \frac{d^n k}{(2\pi)^n} \frac{\Delta n^* \cdot k}{[n \cdot k\, n^* \cdot k + i\eta]^2} \cdot \frac{1}{k^2} \times$$
$$\times [n_\beta (p - 2k)_\varrho + \delta_{\varrho\beta} (n \cdot k - 2p \cdot n) + n_\varrho (k + p)_\beta] \times$$
$$\times \frac{1}{(p-k)^2} \{\delta_{\alpha\beta} - \frac{n^* (p - k)}{n^* \cdot (p - k)\, n (p - k) + i\eta} \times$$
$$\times [n_\alpha (p - k)_\beta + n_\beta (p - k)_\alpha]\}$$

and

$$\Gamma'(2) = \eta g^2 C_{YM} \delta_{mn} \int \frac{d^n k}{(2\pi)^n} \frac{\Delta n^* \cdot k\, n_\alpha}{[n \cdot k\, n^* \cdot k + i\eta]^2} \cdot \frac{1}{k^2} \times$$
$$\times \frac{n^* (p - k)\, n_\varrho}{n^* (p - k)\, n (p - k) + i\eta}. \tag{16}$$

Let us choose a frame in which

$$n = (1, 0, 0, 1), \qquad n^* = (1, 0, 0, -1) \tag{17}$$

and make a Wick rotation $k_0 \to i k_4$. Then we write

$$k_\mu = (k_4; 0, 0, k_3) + K_\mu,$$

where $K_3 = K_4 = 0$ and rescale

$$k_3 \to \eta^{1/2} k_3, \qquad k_4 \to \eta^{1/2} k_4.$$

Now, we perform the integration over k_3 and k_4

$$\int dk_3 dk_4 (k_4^2 + k_3^2 - i\eta)^{-2} = \frac{i\pi}{\eta} \tag{18}$$

and after a simple algebra we get

$$\Gamma' = -g^2 C_{YM} \delta_{mn} \pi \int \frac{d^{n-2} K}{(2\pi)^n K^2} \frac{\Delta n^* \cdot K}{(p - K)^2} \times$$
$$\times \{-2p \cdot n [\delta_{\varrho\alpha} - \frac{1}{n \cdot p} (n_\alpha p_\varrho + n_\varrho p_\alpha - n_\alpha K_\varrho - n_\varrho K_\alpha)] +$$
$$+ n_\varrho [2K_\alpha + \frac{1}{n \cdot p} n_\alpha (K^2 - p^2)]\}. \tag{19}$$

However, here $1/K^2$ and $1/(p - K)^2$ are still in n-dimensional space. First, we have to set them into $(n - 2)$-dimensional space, which means that

$$\frac{1}{K^2} = -\frac{1}{\vec{K}^2},$$

$$\frac{1}{(p-K)^2} = \frac{1}{p_+p_- - (\vec{P} - \vec{K})^2},$$

$$\Delta\vec{n}^* \cdot \vec{P} = -\Delta n^* \cdot p,$$

$$P^2 = -\vec{P}^2. \tag{20}$$

Here p_+ and p_- are Mandelstam's variables

$$p_+ = n \cdot p,$$

$$p_- = n^* \cdot p.$$

It means that in $(n - 2)$-dimensional space

$$\Gamma' = -g^2 C_{YM} \delta_{mn} \frac{\pi}{(2\pi)^n} \Delta n^*_\mu \int \mathrm{d}^{n-2}K \left\{ \frac{K_\mu}{p_+p_- - (\vec{P} - \vec{K})^2} \cdot \frac{n_\varrho p_\alpha}{n \cdot p} - \right.$$

$$- \frac{K_\mu}{\vec{K}^2} \frac{1}{p_+p_- - (\vec{P} - \vec{K})^2} \left[-2p \cdot n \delta_{\varrho\alpha} - \frac{p^2}{n \cdot p} n_\varrho n_\alpha + 2n_\alpha p_\varrho + \right.$$

$$\left.\left. + 2n_\varrho p_\alpha - 2n_\alpha K_\varrho \right] \right\}. \tag{21}$$

To evaluate Γ', we need only three different integrals:

(1) $$\int \mathrm{d}^{n-2}K \frac{K_\mu}{\vec{K}^2 - 2\vec{P} \cdot \vec{K} + \vec{P}^2 - p_+p_-} = P_\mu \left\{ \Gamma\left(\frac{4-n}{2}\right) - \ln(-p_+p_-) \right\},$$

(2) $$\int \mathrm{d}^{n-2}K \frac{K_\mu}{\vec{K}^2 - 2\vec{P} \cdot \vec{K} + \vec{P}^2 - p_+p_-} \cdot \frac{1}{\vec{K}^2} = -\frac{P_\mu}{\vec{P}^2} \ln \frac{p_+p_-}{p^2},$$

(3) $$\int \mathrm{d}^{n-2}K \frac{K_\mu K_\varrho}{\vec{K}^2} \cdot \frac{1}{\vec{K}^2 - 2\vec{P} \cdot \vec{K} + \vec{P}^2 - p_+p_-} = \frac{1}{2} \delta_{\mu\varrho} \Gamma\left(\frac{4-n}{2}\right) +$$

$$+\frac{1}{2}\,\delta_{\mu\varrho}\left[2-\ln(-p_+p_-)\right]-\frac{1}{2}\,\delta_{\mu\varrho}\,\frac{p^2}{\vec{P}^2}\ln\frac{p_+p_-}{p^2}+P_\mu P_\varrho\left\{-\frac{1}{\vec{P}^2}+\right.$$

$$\left.+\frac{p^2}{\vec{P}^4}\ln\frac{p_+p_-}{p^2}\right\}. \tag{22}$$

Owing to $\Delta n^*_\mu P_\mu = -\Delta n^* p$, Γ' becomes

$$\Gamma'\cdot C^{-1}=-\Delta n^*p\left\{\frac{n_\varrho n_\alpha}{n\cdot p}\left[\Gamma\left(\frac{4-n}{2}\right)-\ln(-p_+p_-)\right]\right\}-$$

$$-\frac{\Delta n^*p}{\vec{P}^2}\ln\frac{p_+p_-}{p^2}\left[-2p\cdot n\,\delta_{\varrho\alpha}-\frac{p^2}{n\cdot p}\,n_\varrho n_\alpha+2n_\alpha p_\varrho+2n_\varrho p_\alpha\right]+$$

$$+n_\alpha\Delta n^*_\varrho\left[\Gamma\left(\frac{4-n}{2}\right)+2-\ln(-p_+p_-)-\frac{p^2}{\vec{P}^2}\ln\frac{p_+p_-}{p^2}\right]-$$

$$-2n_\alpha\frac{\Delta n^*p}{\vec{P}^2}\left(-1+\frac{p^2}{\vec{P}^2}\ln\frac{p_+p_-}{p^2}\right)\left(p_\varrho-\frac{1}{2}\,n^*_\varrho p_+-\frac{1}{2}\,n_\varrho p_-\right),$$

where

$$C=g^2C_{YM}\delta_{mn}\cdot i\pi^{n/2}(2\pi)^{-n}. \tag{23}$$

We can write

$$\ln(-p_+p_-)=-\ln(-p^2)+\ln\frac{p^2}{p_+p_-} \tag{24}$$

and after a simple algebra we get

$$\Gamma'C^{-1}=-\left[\Gamma\left(\frac{4-n}{2}\right)+2-\ln(-p^2)+\frac{p_+p_-}{\vec{P}^2}\ln\frac{p^2}{p_+p_-}\right]\times$$

$$\times\left(\Delta n^*p\,\frac{n_\alpha n_\varrho}{n\cdot p}-n_\alpha\Delta n^*_\varrho\right)+\frac{\Delta n^*p}{\vec{P}^2}\left[2n_\alpha p_\varrho-n_\alpha n^*_\varrho p_+-n_\alpha n_\varrho p_-+2\vec{P}^2\,\frac{n_\alpha n_\varrho}{n\cdot p}\right]-$$

$$-\frac{\Delta n^*p}{\vec{P}^2}\ln\frac{p_+p_-}{p^2}\left[-2p\cdot n\,\delta_{\varrho\alpha}-\frac{2p^2}{n\cdot p}\,n_\varrho n_\alpha+2n_\alpha p_\varrho+2n_\varrho p_\alpha+\frac{p^2}{\vec{P}^2}\left(2n_\alpha p_\varrho-\right.\right.$$

$$\left.\left.-n_\alpha n^*_\varrho p_+-n_\alpha n_\varrho p_-+2\vec{P}^2\,\frac{n_\alpha n_\varrho}{n\cdot p}\right)\right].$$

We want to check Eq. (15) which is the Piguet-Sibold equation for the gluon self-energy. With our Γ', the right-hand side of Eq. (15) becomes

$$(p_\mu p_\alpha - \delta_{\alpha\mu}\, p^2)\left\{-2n_\alpha p_\varrho + n_\alpha n_\varrho^* p_+ + n_\alpha n_\varrho p_- - 2\vec{P}^2\,\frac{n_\alpha n_\varrho}{p_+}\right\} +$$

$$+ (\mu \leftrightarrow \varrho) = R_{\mu\varrho} - \frac{4T_{\mu\varrho}}{p_+},$$

$$(p_\mu p_\alpha - \delta_{\alpha\mu}\, p^2)\left(\frac{n_\varrho n_\alpha}{n \cdot p}\,\Delta n^* p - n_\alpha\,\Delta n_\varrho^*\right) + (\mu \leftrightarrow \varrho) = -\frac{1}{p_+}\,\Delta R_{\mu\varrho},$$

$$(p_\mu p_\alpha - \delta_{\alpha\mu}\, p^2)\,[2p \cdot n\,\delta_{\varrho\alpha} + \frac{2p^2}{n \cdot p}\, n_\varrho n_\alpha - 2n_\alpha p_\varrho - 2n_\varrho p_\alpha] +$$

$$+ (\varrho \leftrightarrow \mu) = -\frac{4p^2}{p_+}\, S_{\mu\varrho}, \tag{25}$$

where

$$R_{\mu\nu} = 2p^2 p_- n_\mu n_\nu + p_+^2\,(p_\mu n_\nu^* + p_\nu n_\mu^*) - p_+ p_-\,(p_\mu n_\nu + p_\nu n_\mu) -$$

$$- p^2 p_+\,(n_\mu n_\nu^* + n_\nu n_\mu^*),$$

$$S_{\mu\nu} = p^2 n_\mu n_\nu + p_+^2 \delta_{\mu\nu} - p_+\,(p_\mu n_\nu + p_\nu n_\mu),$$

$$T_{\mu\nu} = (p^2)^2\, n_\mu n_\nu + p_+^2 p_\mu p_\nu - p^2 p_+\,(p_\mu n_\nu + p_\nu n_\mu),$$

$$Q_{\mu\nu} = p^2 \delta_{\mu\nu} - p_\mu p_\nu. \tag{26}$$

Finally, the r. h. s. of Eq. (15) is

$$(p_\mu p_\alpha - \delta_{\alpha\mu} p^2)\,\Gamma'_{\alpha\nu} + (\mu \leftrightarrow \nu) = \frac{1}{p_+}\,\Delta R_{\mu\nu}\left[\Gamma\left(\frac{4-n}{2}\right) + 2 - \ln(-p^2) +\right.$$

$$\left. + \frac{p_+ p_-}{p_+ p_- - p^2}\ln\frac{p^2}{p_+ p_-}\right] + \frac{\Delta n^* p}{p_+ p_- - p^2}\,[-R_{\mu\nu} + \frac{4}{p_+}\,T_{\mu\nu}] -$$

$$- \frac{\Delta n^* p}{(p_+ p_- - p^2)^2} \cdot p^2 \ln\frac{p_+ p_-}{p^2}\,[-R_{\mu\nu} + \frac{4}{p_+}\,T_{\mu\nu}] -$$

$$- \frac{\Delta n^* p}{p_+ p_- - p^2}\ln\frac{p_+ p_-}{p^2} \cdot \frac{4p^2}{p_+}\, S_{\mu\nu}. \tag{27}$$

3. *Complete* $\pi_{\mu\nu}$ *to one-loop order*

In Eq. (34) of Ref. 7 we evaluated the complete self-energy tensor to one-loop order. The graph is shown in Fig. 2.

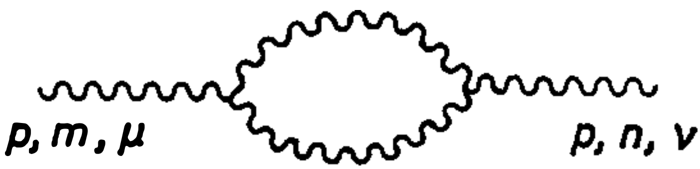


Fig. 2. The gluon self-energy.

It is

$$\pi_{\mu\nu} C^{-1} = \frac{11}{3}\left[\Gamma\left(\frac{4-n}{2}\right) + \frac{67}{33} - \ln(-p^2)\right] Q_{\mu\nu} + \frac{1}{p_+} R_{\mu\nu} \left[\Gamma\left(\frac{4-n}{2}\right) + \right.$$

$$\left. + 2 - \ln(-p^2) + \frac{p_+p_-}{p_+p_- - p^2} \ln \frac{p^2}{p_+p_-}\right] - \frac{4p_-}{p_+(p_+p_- - p^2)} \ln \frac{p^2}{p_+p_-} T_{\mu\nu} +$$

$$+ \frac{4p_-}{p_+} p^2 F(p^2, p_+p_-) S_{\mu\nu}, \tag{28}$$

where

$$F = \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y\, (1 - y + xy)^{-1} [-p_+p_- + y(p_+p_- - p^2)]^{-1} \doteq$$

$$\doteq \int_0^1 \mathrm{d}y\, y^{-1} \ln(1-y)\, [p_+p_- - y(p_+p_- - p^2)]^{-1}. \tag{29}$$

The change in Δn^* in F is

$$\Delta F = \frac{\partial F}{\partial p_+p_-} \cdot p_+ \Delta p_- = \frac{\partial F}{\partial p_+p_-} p \cdot n\, p \cdot \Delta n^*. \tag{30}$$

We introduce the variable

$$\lambda = \frac{p_+p_-}{p_+p_- - p^2}. \tag{31}$$

In fact, we want

$$\frac{\partial}{\partial p_+ p_-}\,[p_+ p_-\, F(p^2, p_+ p_-)] = \frac{\partial}{\partial p_+ p_-}\left\{\int\limits_0^1 \mathrm{d}y \ln(1-y)\left[\frac{1}{y}+\frac{1}{\lambda - y}\right]\right\} =$$

$$= p^2 (p_+ p_-)^{-2} \lambda^{-2} \int\limits_0^1 \mathrm{d}y \ln(1-y)(\lambda - y)^{-2}. \qquad (32)$$

This we integrate by parts to get

$$(p_+ p_- - p^2)^{-1} \ln \frac{p^2}{p_+ p_-}.$$

Now it is trivial to check that the change in Δn^* of $\pi_{\mu\nu}$ in Eq. (28) is identical to Eq. (27).

In a similar way we can verify the Piguet-Sibold equation for a change in Δn.

4. Conclusion

We have verified that the dependence of the gluon self-energy on n^* is consistent with the Piguet-Sibold identity to one-loop order. We have studied ultraviolet divergent parts as well as the finite parts contributing to $\pi_{\mu\nu}$ and Γ'.

Acknowledgement

The author wishes to thank Prof. J. C. Taylor for his invaluable advice.

References

1) J. Scherk and J. H. Schwarz, Gen. Rel. Grav. **6** (1975) 537;
2) S. Mandelstam, Nucl. Phys. **B213** (1983) 149;
3) G. Leibbrandt, Phys. Rev. **D29** (1984) 1699;
4) O. Piguet and K. Sibold, Nucl. Phys. **B253** (1985) 517;
A. Andraši and J. C. Taylor, DAMTP 86-11 preprint, unpublished (1986);
5) T. Zinn-Justin, in Lecture Notes in Physics **37** (1975), ed. T. Ehlers et al. (Berlin, Springer Verlag);
6) A. Andraši and J. C. Taylor, Nucl. Phys. **B302** (1988) 123;
7) A. Andraši and J. C. Taylor, Nucl. Phys. **B310** (1988) 222.

VLASTITA ENERGIJA GLUONA U BAŽDARNOM UVJETU SVJETLOSNOG KONUSA I PIGUET-SIBOLDOV IDENTITET

ANĐELKA ANDRAŠI

Institut »Ruđer Bošković«, 41001 Zagreb

UDK 539.12.01

Originalni znanstveni rad

Razmatrana je ovisnost potpunog tenzora vlastite energije gluona o drugom svjetlosnom vektoru u baždarnom uvjetu svjetlosnog konusa. Verificirani su Piguet-Siboldovi identiteti.